Int. J. Heat Mass Transfer. Vol. 18, pp. 797–804. Pergamon Press 1975. Printed in Great Britain

EXACT SOLUTION OF TEMPERATURE AND MOISTURE DISTRIBUTIONS IN A POROUS HALF-SPACE WITH MOVING EVAPORATION FRONT

M. D. MIKHAILOV
Applied Mathematics Centre, VMEI, Sofia, Bulgaria

(*Received* 16 *November* 1974)

Abstract—Exact solutions are obtained for temperature and moisture distribution as well as the position of the moving evaporation front in a porous half-space. Two mathematical models, corresponding to the drying of the moist body in the period of decreasing rate and the intensive drying in the presence of molar transfer in the region of evaporation are considered. It is shown that the problem solved in [1] is a very special case of the solution presented here. The influence of some of the nondimensional parameters is illustrated by examples.

NOMENCLATURE

A, A_{ji}, A_p, constants of integration;
a_p, molar diffusivity;
a_{qi}, thermal diffusivity;
a_{mi}, moisture diffusivity;
B, B_{ji}, B_p, constants of integration;
c_{qi}, specific heat capacity;
c_{mi}, specific mass capacity;
Fo, Fo_x, Fourier numbers, $a_{q2}\tau/l^2$ and $a_{q2}\tau/x^2$ respectively;
k_i, k_p, thermal and molar conductivity respectively;
Ko_i, Kossovitch number, $r(c_{mi}/c_{qi})(\theta_0-\theta_s)/(t_s-t_0)$;
l, the characteristic length;
Lu_i, Lu_p, Luikov numbers, a_{mi}/a_{qi} and a_p/a_{q2} respectively;
$p(x,\tau)$, pressure;
$P(X, Fo)$, nondimensional pressure, $(p(x,\tau)-p_s)/p_s$;
Pn_i, Posnov number, $\delta_i(t_s-t_0)/(\theta_0-\theta_s)$;
r, latent heat of vaporisation of liquid per unit time;
$s(\tau)$, position of evaporation front;
S, nondimensional position of evaporation front, s/l;
$t_i(x,\tau)$, temperature;
$T_i(X, Fo)$, nondimensional temperature, $(t_i(x,\tau)-t_0)/(t_s-t_0)$;
x, length coordinate;
X, nondimensional length, x/l;
$Z_{ji}(X, Fo)$, potentials defined by equation (2.15);
erf( ), error function;
erfc( ), complimentary error function.

Greek symbols

δ_i, thermal gradient coefficient;
ε_i, phase change criterion;
$\theta_i(x,\tau)$, mass-transfer potential;
$\Theta_i(X, Fo)$, nondimensional mass-transfer potential, $(\theta_0-\theta_i(x,\tau))/(\theta_0-\theta_s)$;
λ, nondimensional constant, $S/(2\sqrt{Fo})$;
ν_p, $\rho_m a_{q2}(1-\varepsilon_2)/(k_p p_s)$;
ν_t, $\rho_m a_{q2}(r/k_1)(\varepsilon_1-\varepsilon_2)/(t_s-t_0)$;
ρ_{mi}, density of moisture per unit volume;
ν_{ji}, ν_j, constants defined by equations (2.16) and (3.27);
$\varphi_{li}(\lambda)$, defined by equation (2.30);
τ, time.

Subscripts

i, j, 1 or 2;
$i = 1$, first region, $0 < x < s$; $1/(4\lambda^2) < Fo_x < \infty$;
$i = 2$, second region, $s < x < \infty$; $0 < Fo_x < 1/(4\lambda^2)$;
s, at surface $x = 0$;
v, vaporising state;
21, 12, ratio of properties of region 2 to 1 and 1 to 2 respectively.

1. INTRODUCTION

AS FAR as 1929 Sherwood pointed out that in the process of drying in the period of decreasing rate a gradual deepening of the evaporation region inside body is observed [2]. Still at that time, on the basis of numerous experiments [3–7] Luikov found out the mechanism of this phenomenon.

He showed that evaporation takes place not only in the moving evaporation front, but all over the boundary region. This is due to the fact that the capillary moisture is removed comparatively easy on the evaporation surface which is an analogue of the freezing boundary in Stephen's problem, while the adsorption moisture, which is strongly attached, is removed by gradual evaporation in the whole region of evaporation. Some more detailed information the reader can find in the well known monograph [2].

The mathematical formulation of the problem of determining the moisture and temperature fields in the presence of a deepening region of evaporation is given in [2]. But up to now exact analytical solutions of the problem were not announced. In connection with this the approximate solution obtained by heat balance integral technique in [1] is of considerable interest.

Here are derived the exact solutions of two more complicated models. More convenient charts for temperature and moisture are proposed and influence of some nondimensional parameters is illustrated.

Evidently there is contradiction between the simplifying assumptions accepted in [1] and [8]. Gupta neglected the second term in the r.h.s. of his equation (2.4) while Bruin—the first term. But as we have shown in [9], quite a complex mechanism is hidden behind the exterior simplicity of the process of drying which should be studied only on the basis of Luikov's system without any simplifications of the latter.

2. DRYING IN THE PERIOD OF DECREASING RATE

Let us consider the flow of heat and moisture through a porous half-space ($x > 0$) during drying. It has been pointed out [3–7] that the evaporation front moves forward, deepening in the body. Let its position at time τ be given by $x = s(\tau)$. It divides the porous body into two regions, in each one the process of drying being described by Luikov's system. The thermophysical parameters change in a jumplike manner when crossing the evaporation front, which reduces the problem to the solution of the following equations [2]:

$$\frac{\partial t_i(x,\tau)}{\partial \tau} = a_{qi}\frac{\partial^2 t_i(x,\tau)}{\partial x^2} + \varepsilon_i r\frac{c_{mi}}{c_{qi}}\frac{\partial \theta_i(x,\tau)}{\partial \tau} \tag{2.1}$$

$$\frac{\partial \theta_i(x,\tau)}{\partial \tau} = a_{mi}\frac{\partial^2 \theta_i(x,\tau)}{\partial x^2} + a_{mi}\delta_i\frac{\partial^2 t_i(x,\tau)}{\partial x^2} \tag{2.2}$$

where the subscript $i = 1$ corresponds to the evaporation region $0 < x < s(\tau)$ and $i = 2$ to $s(\tau) < x < \infty$.

The initial distribution of temperature and moisture are uniform

$$t_2(x,0) = t_2(\infty,\tau) = t_0, \quad \theta_2(x,0) = \theta_2(\infty,\tau) = \theta_0. \tag{2.3}$$

It is also assumed that on the surface of the half-space the temperature and moisture are constant but differing from the initial ones

$$t_1(0,s) = t_s, \quad \theta_1(0,s) = \theta_s. \tag{2.4}$$

These conditions do not correspond to the process of drying, but they allow to obtain an exact analytical solution.

On the evaporation front there exists an equality between temperatures and mass-transfer potentials

$$t_1(s,\tau) = t_2(s,\tau) = t_v(\tau), \quad \theta_1(s,\tau) = \theta_2(s,\tau) = \theta_v. \tag{2.5}$$

The jumplike change of the phase change number from ε_2 to at ε_1 takes place at a certain humidity θ_v, which being available, leads to the deepening of the evaporation surface [10]. In contrast to Stephan's problem of freezing a moist body, the evaporation surface temperature $t_v(\tau)$ appears to be a variable. A characteristic quantity for the evaporation surface turns out to be a mass content below which the mass supply is lower than the removal of vapours from the evaporation surface and reasons in deepening of the latter [10].

An interface condition concerns the heat flux required to evaporate the moisture at this evaporation front. As it moves forward at a distance ds, a quantity of heat per unit area is necessary to evaporate the moisture at this surface, which yields

$$\sum_{i=1}^{2}(-1)^i k_i\frac{\partial t_i(s,\tau)}{\partial x} = r(\varepsilon_1-\varepsilon_2)\rho_{m2}\frac{\mathrm{d}s}{\mathrm{d}\tau}. \tag{2.6}$$

The moisture balance at the evaporation front gives

$$\sum_{i=1}^{2}(-1)^i a_{mi}\left\{\frac{\partial \theta_i(s,\tau)}{\partial x} + \delta_i\frac{\partial t_i(s,\tau)}{\partial x}\right\} = 0. \tag{2.7}$$

The set of equations (2.1)–(2.2) can be given the nondimensional form as

$$\frac{\partial T_i(X,Fo)}{\partial Fo} = a_{i2}\frac{\partial^2 T_i(X,Fo)}{\partial X^2} - \varepsilon_i Ko_i\frac{\partial \Theta_i(X,Fo)}{\partial Fo} \tag{2.8}$$

$$\frac{\partial \Theta_i(X,Fo)}{\partial Fo} = a_{i2}Lu_i\left\{\frac{\partial^2 \Theta_i(X,Fo)}{\partial X^2} - Pn_i\frac{\partial^2 T_i(X,Fo)}{\partial X^2}\right\}. \tag{2.9}$$

The initial and boundary conditions are

$$T_2(X,0) = T_2(\infty,Fo) = 0, \quad \Theta_2(X,0) = \Theta_2(\infty,Fo) = 0 \tag{2.10}$$

$$T_1(0,Fo) = 1, \quad \Theta_1(0,Fo) = 1. \tag{2.11}$$

The interface conditions are

$$T_1(S,Fo) = T_2(S,Fo) = T_v(Fo), \quad \Theta_1(S,Fo) = \Theta_2(S,Fo) = \Theta_v \tag{2.12}$$

$$\sum_{i=1}^{2}(-1)^i k_{i1}\frac{\partial T_i(S,Fo)}{\partial X} = v_t\frac{\mathrm{d}S(Fo)}{\mathrm{d}Fo} \tag{2.13}$$

$$\sum_{i=1}^{2}(-1)^i a_{i2}Lu_i\left\{\frac{\partial \Theta_i(S,Fo)}{\partial X} - Pn_i\frac{\partial T_i(S,Fo)}{\partial X}\right\} = 0. \tag{2.14}$$

In [11] it is shown that the systems (2.8)–(2.9) can be transformed into the decoupled equations

$$v_{ji}^2\frac{\partial Z_{ji}(X,Fo)}{\partial Fo} = a_{i2}\frac{\partial^2 Z_{ji}(X,Fo)}{\partial X^2} \tag{2.15}$$

where

$$v_{ji}^2 = \frac{1}{2}\left\{1+\varepsilon_i Ko_i Pn_i + \frac{1}{Lu} + (-1)^j \times\sqrt{\left[\left(1+\varepsilon_i Ko_i Pn_i+\frac{1}{Lu}\right)^2 - \frac{4}{Lu}\right]}\right\}. \tag{2.16}$$

The pure heat-conduction type differential equations, like (2.15) have the following solution

$$Z_{ji}(X,Fo) = A_{ji} + B_{ji}\,\mathrm{erf}\left(\frac{v_{ji}X}{2\sqrt{(a_{i2}Fo)}}\right). \tag{2.17}$$

The potentials $T_i(X, Fo)$ and $\Theta_i(X, Fo)$ are given from the linear combinations of $Z_{ji}(X, Fo)$ [11]:

$$T_i(X, Fo) = \frac{1}{v_{2i}^2 - v_{1i}^2} \sum_{j=1}^{2} (-1)^i (1 - v_{3-j,i}^2) \times \left[A_{ji} + B_{ji} \operatorname{erf}\left(\frac{v_{ji} X}{2\sqrt{(a_{i2} Fo)}} \right) \right] \quad (2.18)$$

$$\Theta_i(X, Fo) = \frac{Pn_i}{v_{2i}^2 - v_{1i}^2} \sum_{j=1}^{2} (-1)^j \times \left[A_{ji} + B_{ji} \operatorname{erf}\left(\frac{v_{ji} X}{2\sqrt{(a_{i2} Fo)}} \right) \right] \quad (2.19)$$

where A_{ji} and B_{ji} are constants which are to be chosen to satisfy the initial and boundary conditions. For the case discussed here this is possible and consequently the problem has exact analytical solution.

A system of two algebraic equations is obtained from the initial conditions (2.10) and its solution yields

$$B_{j2} = -A_{j2}, \quad j = 1, 2. \quad (2.20)$$

In an analogous way from the boundary conditions (2.11):

$$A_{j1} = 1 + \frac{v_{j1}^2 - 1}{Pn_1}, \quad j = 1, 2. \quad (2.21)$$

Substituting the solutions (2.18) and (2.19) in the conditions (2.12), using (2.20) and (2.21) and having in mind that $(v_{ji}^2 - 1)(v_{3-j,i}^2 - 1) = -\varepsilon_i Ko_i Pn_i$ one obtains the system from which it follows that

$$B_{j1} = \frac{T_v - 1 + (\Theta_v - 1)(v_{j1}^2 - 1)/Pn_1}{\operatorname{erf}(v_{j1} \lambda/\sqrt{a_{12}})} \quad (2.22)$$

and

$$A_{j2} = \frac{T_v + \Theta_v (v_{j2}^2 - 1)/Pn_2}{\operatorname{erfc}(v_{j2} \lambda)} \quad (2.23)$$

where

$$\lambda = s/(2\sqrt{Fo}) \quad \text{and} \quad T_v = \text{const}.$$

Substituting (2.20)–(2.23) in the solutions (2.18)–(2.19) after some mathematical operations one has the following final results:

$$T_1(Fo_x) = 1 + \frac{1}{v_{21}^2 - v_{11}^2} \sum_{j=1}^{2} (-1)^j \times [(1 - v_{3-j,1}^2)(T_v - 1) + \varepsilon_1 Ko_1(\Theta_v - 1)] \times \operatorname{erf}\left(\frac{v_{j1}}{2\sqrt{(a_{12} Fo_x)}} \right) \Big/ \operatorname{erf}(v_{j1} \lambda/\sqrt{a_{12}}) \quad (2.24)$$

$$\Theta_1(Fo_x) = 1 + \frac{1}{v_{21}^2 - v_{11}^2} \sum_{j=1}^{2} (-1)^j \times [Pn_1(T_v - 1) + (v_{j1}^2 - 1)(\Theta_v - 1)] \times \operatorname{erf}\left(\frac{v_{j1}}{2\sqrt{(a_{12} Fo_x)}} \right) \Big/ \operatorname{erf}(v_{j1} \lambda/\sqrt{a_{12}}) \quad (2.25)$$

where $Fo_x \geq 1/(4\lambda^2)$ and

$$T_2(Fo_x) = \frac{1}{v_{22}^2 - v_{12}^2} \sum_{j=1}^{2} (-1)^j \times [(1 - v_{3-j,2}^2) T_v + \varepsilon_2 Ko_2 \Theta_v] \times \operatorname{erfc}\left(\frac{v_{j2}}{2\sqrt{Fo_x}} \right) \Big/ \operatorname{erfc}(v_{j2} \lambda) \quad (2.26)$$

$$\Theta_2(Fo_x) = \frac{1}{v_{22}^2 - v_{12}^2} \sum_{j=1}^{2} (-1)^j [Pn_2 T_v + (v_{j2}^2 - 1)\Theta_v] \times \operatorname{erfc}\left(\frac{v_{j2}}{2\sqrt{Fo_x}} \right) \Big/ \operatorname{erfc}(v_{j2} \lambda) \quad (2.27)$$

where $Fo_x \leq 1/(4\lambda^2)$.

With the help of these solutions from the boundary conditions (2.13) and (2.14) one gets:

$$[(\sqrt{a_{21}})\varphi_{11} + k_{21}\varphi_{12}] T_v + [(\sqrt{a_{21}})\varepsilon_1 Ko_1 \varphi_{21} + k_{21} \varepsilon_2 Ko_2 \varphi_{22}] \Theta_v = (\sqrt{a_{21}})[\varphi_{11} + \varepsilon_1 Ko_1 \varphi_{21}] - (\sqrt{\pi}) v_t \lambda \quad (2.28)$$

$$[(\sqrt{a_{12}}) Lu_1 Pn_1(\varphi_{11} - \varphi_{21}) + Lu_2 Pn_2(\varphi_{12} - \varphi_{22})] T_v + [(\sqrt{a_{12}}) Lu_1(\varphi_{31} + \varepsilon_1 Ko_1 Pn_1 \varphi_{21}) + Lu_2(\varphi_{32} + \varepsilon_2 Ko_2 Pn_2 \varphi_{22})] \Theta_v = (\sqrt{a_{12}}) Lu_1 [Pn_1(\varphi_{11} - \varphi_{21}) + \varphi_{31} + \varepsilon_1 Ko_1 Pn_1 \varphi_{21}] \quad (2.29)$$

where

$$\varphi_{l,i} = \frac{1}{v_{2i}^2 - v_{1i}^2} \sum_{j=1}^{2} (-1)^j v_{ji} \times [1 + (1 - l/2)((l-3) v_{3-j,i}^2 + (l-1) v_{ji}^2)] \times \exp[-(v_{ji}\lambda/(\sqrt{a_{i2}}))^2] / [(2-i)\operatorname{erf}(v_{j1}\lambda/\sqrt{a_{12}}) + (i-1)\operatorname{erfc}(v_{j2}\lambda)] \quad (2.30)$$

where $l = 1, 2, 3$; $i = 1, 2$.

When Θ_v are known from equations (2.28)–(2.29), using a computer, one can easily obtain T_v and λ, which leads to obtaining the exact analytical solutions (2.24)–(2.27) to analyze the process of drying with a deepening region of evaporation.

As it is seen from the solution presented, on the surface of the evaporation front not only mass-transfer potential but also temperature are constant. Perhaps this is a result from the boundary conditions (2.11).

In the case that the moisture in region $i = 1$ is in vapour form only, that is $\varepsilon_1 = 1$, $Ko_1 = 0$, $Pn_1 = 0$, $Lu_1 = \infty$, from (2.16) and (2.29) follows that $v_{21}^2 = 1$, $v_{11}^2 = 0$, $\Theta_v = 1$. Then the solutions (2.24)–(2.25) take the form

$$T_1(Fo_x) = 1 + (T_v - 1)\operatorname{erf}\left(\frac{1}{2\sqrt{(a_{12} Fo_x)}} \right) \Big/ \operatorname{erf}(\lambda/\sqrt{a_{12}}); \quad \Theta_1(Fo_x) = 1 \quad (2.31)$$

and equation (2.28) yields the following transcendental equation

$$(T_v - 1)\exp[-(\lambda/\sqrt{a_{12}})^2]/\operatorname{erf}(\lambda/\sqrt{a_{12}}) + \frac{k_{21}\sqrt{a_{21}}}{v_{22}^2 - v_{12}^2} \times \sum_{j=1}^{2} (-1)^j v_{j2} [(1 - v_{3-j,2}^2) T_v + \varepsilon_2 Ko_2] \times \exp[-(v_{j2}\lambda)^2]/\operatorname{erfc}(v_{j2}\lambda) + \sqrt{(\pi a_{12})} v_t \lambda = 0. \quad (2.32)$$

This particular case is obtained when the vapour is not subject to considerable resistance in its movement in the region of evaporation, and therefore at the front there is constant pressure. It is well known that at a fixed pressure for every liquid there exists a temperature at which it evaporates completely. That is why the temperature T_v turns out to be a known and the transcendental equation (2.32) fully determines λ. The exact solution of Gupta's case [1] is easily obtained as a special case from equations (2.31), (2.26), (2.27) and (2.32) when one has: $a_{21} = 1$, $Pn_2 = 0$, $v_{22}^2 = 1$ and $v_{21}^2 = 1/Lu_2$.

3. DRYING WITH MOLAR TRANSFER

In [1] is discussed the case when in the boundary region $0 < x < s(\tau)$ the moisture is in vapour form only. In this region because of the vigorous vaporization takes rise a stable pressure gradient which calls forth a molar transfer of the type of filtration through a porous wall.

A considerably more precise mathematical model of this phenomenon is given by the equations

$$\frac{\partial t_1(x,\tau)}{\partial \tau} = a_{q1}\frac{\partial^2 t_1(x,\tau)}{\partial x^2} \tag{3.1}$$

$$\theta_1(x,\tau) = \theta_v \tag{3.2}$$

$$\frac{\partial p(x,\tau)}{\partial \tau} = a_p\frac{\partial^2 p(x,\tau)}{\partial x^2} \tag{3.3}$$

and

$$\frac{\partial t_2(x,\tau)}{\partial \tau} = a_{q2}\frac{\partial^2 t_2(x,\tau)}{\partial x^2} + \varepsilon_2 r\frac{c_{m2}}{c_{q2}}\frac{\partial \theta_2(x,\tau)}{\partial \tau} \tag{3.4}$$

$$\frac{\partial \theta_2(x,\tau)}{\partial \tau} = a_{m2}\frac{\partial^2 \theta_2(x,\tau)}{\partial x^2} + a_{m2}\delta_2\frac{\partial^2 t_2(x,\tau)}{\partial x^2}. \tag{3.5}$$

where the subscript 1 corresponds to $0 \leqslant x \leqslant s(\tau)$ and 2 to $s(\tau) \leqslant x \leqslant \infty$.

The initial ($\tau = 0$) and boundary ($x = 0$) conditions are stated as follows:

$$t_2(x,0) = t_2(\infty,\tau) = t_0; \quad \theta_2(x,0) = \theta_2(\infty,\tau) = \theta_0 \tag{3.6}$$

$$t_1(0,\tau) = t_s; \quad p(0,\tau) = p_s. \tag{3.7}$$

At the moving evaporation front the conditions are:

$$t_1(s,\tau) = t_2(s,\tau) = f[p(s,\tau)] \tag{3.8}$$

$$\theta_2(s,\tau) = \theta_v \tag{3.9}$$

where $f[p(s,\tau)]$ denotes the relation between boiling temperature and pressure.

Heat and moisture balance at the evaporation front yields:

$$\sum_{i=1}^{2}(-1)^i k_i\frac{\partial t_i(s,\tau)}{\partial x} = r(1-\varepsilon_2)\rho_{m2}\frac{ds}{d\tau} \tag{3.10}$$

$$k_p\frac{\partial p(s,\tau)}{\partial x} = (1-\varepsilon_2)\rho_{m2}\frac{ds}{d\tau}. \tag{3.11}$$

The equations (3.1)–(3.11) can be represented in the nondimensional form as below

$$\frac{\partial T_1(X,Fo)}{\partial Fo} = a_{12}\frac{\partial^2 T_1(X,Fo)}{\partial X^2} \tag{3.12}$$

$$\Theta_1(X,Fo) = 1 \tag{3.13}$$

$$\frac{\partial P(X,Fo)}{\partial Fo} = Lu_p\frac{\partial^2 P(X,Fo)}{\partial X^2} \tag{3.14}$$

$$\frac{\partial T_2(X,Fo)}{\partial X^2} = Lu\frac{\partial^2 T_2(X,Fo)}{\partial X^2} - \varepsilon Ko\frac{\partial \Theta_2(X,Fo)}{\partial Fo} \tag{3.15}$$

$$\frac{\partial \Theta_2(X,Fo)}{\partial Fo} = Lu\left\{\frac{\partial^2 \Theta_2(X,Fo)}{\partial X^2} - Pn\frac{\partial^2 T_2(X,Fo)}{\partial X^2}\right\} \tag{3.16}$$

$$T_2(X,0) = T_2(\infty,Fo) = 0, \quad \Theta_2(X,0) = \Theta_2(\infty,Fo) = 0 \tag{3.17}$$

$$T_1(0,Fo) = 1, \quad P(0,Fo) = 0 \tag{3.18}$$

$$T_1(S,Fo) = T_2(S,Fo) = F[P(S,Fo)] \tag{3.19}$$

$$\Theta_2(S,Fo) = 1 \tag{3.20}$$

$$\sum_{i=1}^{2}(-1)^i k_{2i}\frac{\partial T_i(S,Fo)}{\partial X} = v_t\frac{dS}{dFo} \tag{3.21}$$

$$\frac{\partial P(S,Fo)}{\partial X} = v_p\frac{dS}{dFo} \tag{3.22}$$

where $Lu = Lu_2$, $Pn = Pn_2$, $Ko = Ko_2$ and $\varepsilon = \varepsilon_2$.

The solutions of (3.12)–(3.16), similarly to (2.18)–(2.19) can be written as:

$$T_1(X,Fo) = A + B\,\mathrm{erf}\left(\frac{X}{2\sqrt{(a_{12}Fo)}}\right) \tag{3.23}$$

$$P(X,Fo) = A_p + B_p\,\mathrm{erf}\left(\frac{X}{2\sqrt{(Lu_pFo)}}\right) \tag{3.24}$$

$$T_2(X,Fo) = \frac{1}{v_2^2 - v_1^2}\sum_{j=1}^{2}(-1)^j(1-v_{3-j}^2)\times\left[A_j + B_j\,\mathrm{erf}\left(\frac{v_jX}{2\sqrt{Fo}}\right)\right] \tag{3.25}$$

$$\Theta_2(X,Fo) = \frac{1}{v_2^2 - v_1^2}\sum_{j=1}^{2}(-1)^j\left[A_j + B_j\,\mathrm{erf}\left(\frac{v_jX}{2\sqrt{Fo}}\right)\right] \tag{3.26}$$

where

$$v_j^2 = \frac{1}{2}\left\{1 + \varepsilon Ko Pn + \frac{1}{Lu} + (-1)^j \times\sqrt{\left[\left(1 + \varepsilon Ko Pn + \frac{1}{Lu}\right)^2 - \frac{4}{Lu}\right]}\right\},\ j = 1, 2 \tag{3.27}$$

and A, B, A_p, B_p, A_j, B_j are to be chosen to satisfy the initial and boundary conditions (3.17)–(3.22).

From the initial conditions (3.17) one obtains:

$$B_j = -A_j \tag{3.28}$$

The boundary conditions (3.18) and (3.22) yield:

$$A = 1, \quad A_p = 0 \tag{3.29}$$

$$B_p = \lambda v_p\sqrt{(\pi Lu_p)}/\exp(-\lambda^2/Lu_p) \tag{3.30}$$

where $\lambda = S/(2\sqrt{Fo})$.

From (3.19) and (3.20) it follows

$$A + B\,\mathrm{erf}(\lambda/\sqrt{a_{12}}) = \frac{1}{v_2^2 - v_1^2}\sum_{j=1}^{2}(-1)^j(1-v_{3-j}^2)A_j\,\mathrm{erfc}(v_j\lambda) = T_v \tag{3.31}$$

$$\sum_{j=1}^{2}(-1)^jA_j\,\mathrm{erfc}(v_j\lambda) = (v_2^2 - v_1^2)/Pn. \tag{3.32}$$

Then B and A_j may be obtained in term of T_v:

$$B = (T_v - 1)/\mathrm{erf}(\lambda/\sqrt{a_{12}}) \quad (3.33)$$

$$A_j = \left(T_v + \frac{v_j^2 - 1}{Pn}\right)\Big/\mathrm{erfc}(v_j\lambda). \quad (3.34)$$

After substituting (3.29), (3.30), (3.33) and (3.34) in the solutions (3.23)–(3.26) the latter take the form

$$T_1(Fo_x) = 1 + (T_v - 1)\,\mathrm{erf}\left[\frac{1}{2\sqrt{(a_{12}Fo_x)}}\right]\Big/\mathrm{erf}(\lambda/\sqrt{a_{12}}) \quad (3.35)$$

$$P(Fo_x) = \lambda v_p(\pi Lu_p)^{\frac{1}{2}}\,\mathrm{erf}\left[\frac{1}{2\sqrt{(Lu_p Fo_x)}}\right]\Big/\exp(-\lambda^2/Lu_p) \quad (3.36)$$

where $Fo_x \geqslant 1/(4\lambda^2)$ and

$$T_2(Fo_x) = \frac{1}{v_2^2 - v_1^2}\sum_{j=1}^{2}(-1)^j[(1 - v_{3-j}^2)T_v + \varepsilon Ko] \times \mathrm{erfc}\left(\frac{v_j}{2\sqrt{Fo_x}}\right)\Big/\mathrm{erfc}(v_j\lambda) \quad (3.37)$$

$$\Theta_2(Fo_x) = \frac{1}{v_2^2 - v_1^2}\sum_{j=1}^{2}(-1)^j[Pn\,T_v + v_j^2 - 1] \times \mathrm{erfc}\left(\frac{v_j}{2\sqrt{Fo_x}}\right)\Big/\mathrm{erfc}(v_j\lambda) \quad (3.38)$$

where $Fo_x \leqslant 1/(4\lambda^2)$.

The boundary condition (3.21) yields the following transcendental equation for determining λ:

$$(\sqrt{a_{21}})(T_v - 1)\exp[-(\lambda/\sqrt{a_{12}})^2]/\mathrm{erf}(\lambda/\sqrt{a_{12}}) + v_t\lambda\pi^{\frac{1}{2}} + \frac{k_{21}}{v_2^2 - v_1^2}\sum_{j=1}^{2}(-1)^j v_j[\varepsilon Ko + T_v(1 - v_{3-j}^2)] \times \exp[-(v_j\lambda)^2]/\mathrm{erfc}(v_j\lambda) = 0. \quad (3.39)$$

It is obvious that when calculating λ one has to take into account the dependence of the nondimensional temperature upon the nondimensional pressure

$$T_v = F[P(S, Fo)] = F[\lambda v_p(\pi Lu_p)^{\frac{1}{2}}\,\mathrm{erf}(\lambda/\sqrt{Lu_p})/\exp(-\lambda^2/Lu_p)] \quad (3.40)$$

In the case that the porous structure does not resist the vapour flow one has $a_p = 0$ and $k_p = \infty$; that is $v_p = 0$, $Lu_p = \infty$ and hence the solutions (3.35)–(3.39) turn out to be identical to (2.31), (2.26), (2.27) and (2.32).

4. RESULTS FOR SOME PARTICULAR CASES AND DISCUSSIONS

The solutions (3.35)–(3.40) were coded in ALGOL. In this section we discuss the results for the case when the porous structure does not hinder the vapour flow, which is the reason for T_v to be given as a constant.

In Table 1 are presented numerical results for the nondimensional temperature and moisture potentials for the same values of Lu, ε, Ko, v_t, a_{12}, k_{21} and T_v as those in [1]. The identity of data is easily established through the following relations between our quantities (T_v, v_t) and Gupta's ones (v, p):

$$T_v = 1\Big/\left(1 + \frac{k_{21}}{p}\right), \quad v_t = v k_{21}\Big/\left(1 + \frac{p}{k_{21}}\right).$$

Table 1. Nondimensional temperature and mass-transfer potentials for $\varepsilon = 0{\cdot}5$, $Pn = 1$, $Lu = 0{\cdot}5$, $Ko = 1{\cdot}2$, $v_t = 5$, $a_{12} = 1$, $k_{21} = 1$, $T_v = 0{\cdot}5$

Fo_x	T	Θ
0·3	0·068	0·016
0·4	0·096	0·055
0·5	0·122	0·101
0·6	0·146	0·147
0·8	0·187	0·234
1	0·221	0·310
1·2	0·250	0·376
1·6	0·294	0·483
2	0·328	0·565
3	0·386	0·708
4	0·423	0·802
5	0·449	0·869
6	0·469	0·920
8	0·498	0·994
8·215	0·500	1·000
10	0·546	
12	0·585	
16	0·640	
20	0·678	
30	0·736	
40	0·772	
50	0·796	
60	0·814	
80	0·838	
100	0·855	

In contrast to [1] the value of Posnov number is assumed to be 1 instead of 0 so that account is taken for the influence of the temperature gradient on the moisture movement.

To evaluate the influence of the nondimensional parameters Pn, Lu, ε, Ko, and v_t, the latter were varied as follows:

$Pn = 0$, 0·25, 0·5, 0·75, 1·0 and 1·5;
$Lu = 0{\cdot}05$, 0·1, 0·2, 0·3, 0·4 and 0·5;
$\varepsilon = 0$, 0·1, 0·3, 0·5, 0·7 and 0·9;
$Ko = 0$, 0·4, 0·8, 1·2, 1·6 and 2;
$v_t = 1$, 5, 10, 25 and 50.

The results of the numerical calculations were shown in Figs. 1–4, where one of our figures contains four figures from [1]. If $Fo_x < 0{\cdot}2$ it might happen that $\Theta < 0$; this is not shown on the figures, because the negative values are below 0·02.

The diagrams shown can be interpreted in two different manners:

(a) The figures represent the time changes of temperature and moisture potentials for a fixed space position. At the beginning of the process the temperature gradually rises while from a fixed moment on moisture rapidly evaporates. When the evaporation front reaches the point under consideration the temperature becomes equal to T_v and all the moisture evaporates. Later temperature continues to rise as time goes on, approaching the surface one.

(b) For a fixed moment of time the figures show temperature and moisture distributions in a halfspace. The surface $X = 0$ corresponds to $Fo_x \to \infty$ which is the reason for the right hand side with temperature above T_v to give the temperature distribution in the surface layer where moisture is in vapour form only.

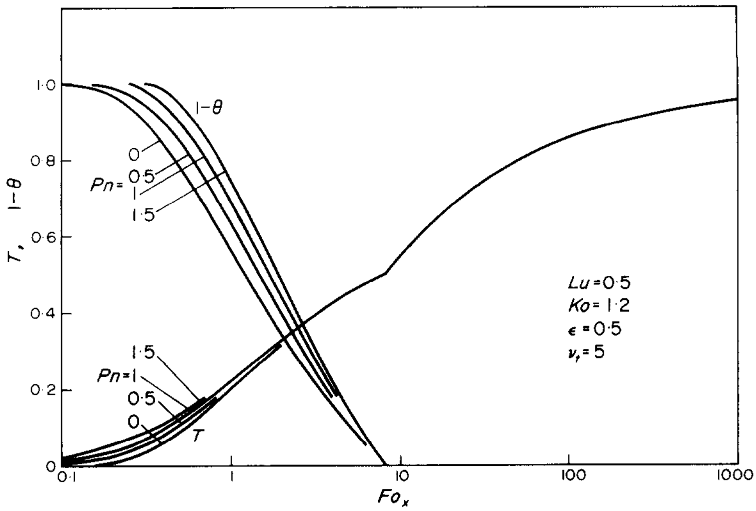

FIG. 1. Effect of variability of Pn on nondimensional temperature and mass-transfer potentials for $\varepsilon = 0{\cdot}5$, $Lu = 0{\cdot}5$, $Ko = 1{\cdot}2$, $v_t = 5$, $a_{12} = 1$, $k_{21} = 1$, $T_v = 0{\cdot}5$.

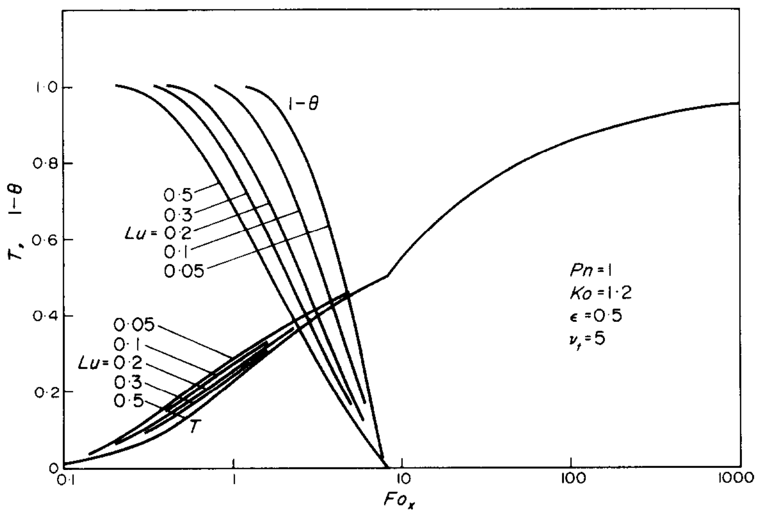

FIG. 2. Effect of variability of Lu on nondimensional temperature and mass-transfer potentials for $\varepsilon = 0{\cdot}5$, $Pn = 1$, $Ko = 1{\cdot}2$, $v_t = 5$, $a_{12} = 1$, $k_{21} = 1$, $T_v = 0{\cdot}5$.

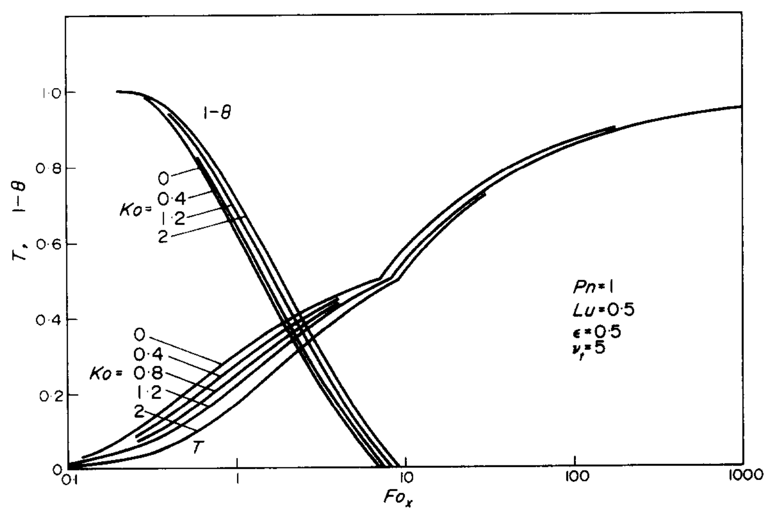

FIG. 3. Effect of variability of Ko on nondimensional temperature and mass-transfer potentials for $\varepsilon = 0{\cdot}5$, $Lu = 0{\cdot}5$, $Pn = 1$, $v_t = 5$, $a_{12} = 1$, $k_{21} = 1$, $T_v = 0{\cdot}5$.

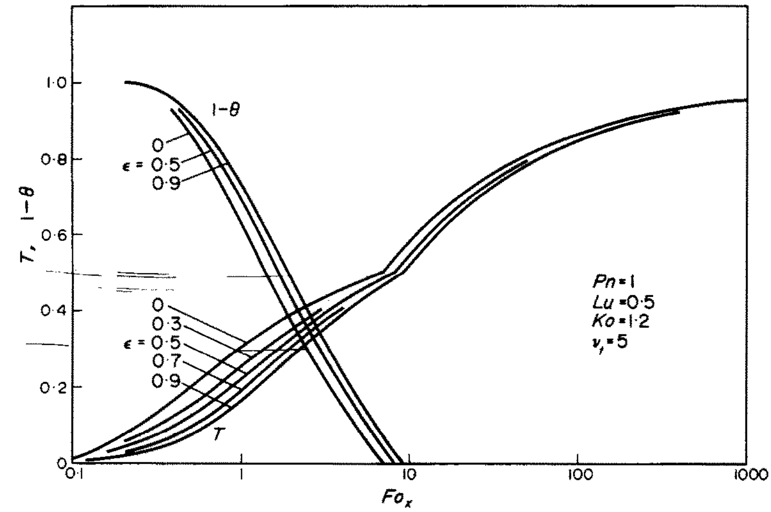

FIG. 4. Effect of variability of ε on nondimensional temperature and mass-transfer potentials for $Lu = 0{\cdot}5$, $Pn = 1$, $Ko = 1{\cdot}2$, $v_t = 5$, $a_{12} = 1$, $k_{21} = 1$, $T_v = 0{\cdot}5$.

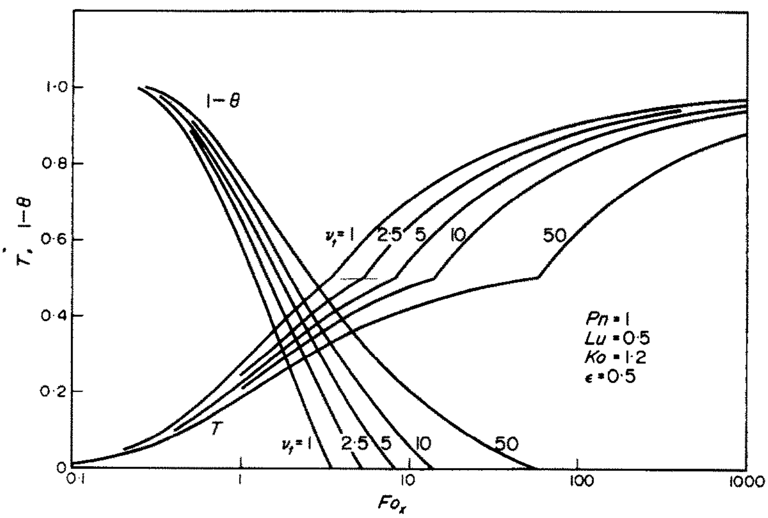

FIG. 5. Effect of variability of ε on nondimensional temperature and mass-transfer potentials for $\varepsilon = 0{\cdot}5$, $Lu = 0{\cdot}5$, $Pn = 1$, $Ko = 1{\cdot}2$, $a_{12} = 1$, $k_{21} = 1$, $T_v = 0{\cdot}5$.

Figure 1 represents the influence of Posnov's number. It can be seen that the position of the evaporation front does not depend on *Pn*. Therefore the Gupta's simplifying assumption [1] that $Pn = 0$ does not reason in considerable inaccuracies.

Figure 2 illustrates the negligible influence of Luikov's number both on the position of the evaporation front and on the temperature distribution. The drying occurs in a region which considerably narrows with the decreasing of *Lu*.

From Fig. 3 it is seen that in comparison to *Pn* and *Lu* the Kossovitch number influences strongly the process. The temperature at a fixed position decreases as *Ko* increases.

Fully analogous is the influence of the phase change criterion (Fig. 4) but nevertheless the influence of ε and *Ko* is considerably weaker than the one shown in Fig. 5 influence of the nondimensional heat of evaporation v_t.

The curves in Fig. 5 confirm all conclusions of Gupta [1] and demonstrate convincingly the decisive influence of v_t in comparison to that of *Pn*, *Lu*, *Ko* and ε.

It may be concluded that v_t characterizes the effect of the deepening of the evaporation front on unsteady state heat and mass transfer in a porous system.

Acknowledgements—The author is highly indebted to the late Prof. Dr. A. V. Luikov for suggesting the problem in a private letter of 10 December 1973. The analytical solutions were reported at the International School (4–19 June 1974) initiated by Prof. A. V. Luikov at the International Center for Heat and Mass Transfer at the Academy of Science of the Socialist Countries.

REFERENCES

1. L. N. Gupta, An approximate solution of the generalized Stefan's problem in a porous medium, *Int. J. Heat Mass Transfer* **17**, 313–321 (1974).

2. A. V. Luikov, *Theory of Drying*. Energiya, Moscow (1968).
3. A. V. Luikov, *Kinetics and Dynamics of Drying and Moisture Processes*. Gizlegprom, Moscow (1938).
4. A. V. Luikov, On the thermodiffusion of moisture, *Zh. Priul. Khim.* **8**, 19–57 (1935).
5. A. V. Luikov, The drying of peat, *Ind. Engng Chem.* **27**, 406 (1935).
6. A. V. Luikov, The maximum allowable moisture gradient, *Trans. Engng Cer. Soc.* **35**, 17 (1935).
7. A. V. Luikov, Geschwindigkeits- und Temperaturkurven des Trocknungsvorgang, *Kolloid Zh.* **71**, 103 (1935).
8. S. Bruin, Calculation of temperature and moisture distribution during contact drying of a sheet of moist material, *Int. J. Heat Mass Transfer* **12**, 45–59 (1969).
9. M. D. Mikhailov and B. K. Shishedjiev, Temperature and moisture distributions during contact drying of a moist porous sheet, *Int. J. Heat Mass Transfer* **18**, 15–27 (1975).
10. A. V. Luikov, Systems of differential equations of heat and mass transfer in capillary porous bodies, *Int. J. Heat Mass Transfer* **18**, 1–17 (1975).
11. M. D. Mikhailov, General solutions of the diffusion equations coupled at boundary conditions, *Int. J. Heat Mass Transfer* **16**, 2155–2164 (1973).

SOLUTIONS EXACTES DE DISTRIBUTION DE TEMPERATURE ET D'HUMIDITE DANS UN DEMI-ESPACE POREUX AVEC FRONT D'EVAPORATION EN MOUVEMENT

Résumé—Des solutions exactes sont obtenues pour les distributions de température et d'humidité ainsi que pour la position du front d'évaporation en mouvement dans un demi-espace poreux.

Deux modèles mathématiques sont considérés, correspondant au séchage d'un corps humide dans la période de diminution de la vitesse d'évaporation et au séchage intensif en présence du transfert de masse dans la région d'évaporation. Il est montré que le problème résolu dans [1] est un cas très spécial de la solution présentée ici. L'influence de quelques uns des paramètres adimensionnels est illustrée par des exemples.

EXAKTE LÖSUNG FÜR DIE TEMPERATUR- UND FEUCHTIGKEITSVERTEILUNG IM PORÖSEN HALBRAUM MIT WANDERNDER VERDAMPFUNGSFRONT

Zusammenfassung—Für einen porösen Halbraum wurden exakte Lösungen sowohl für die Temperatur- und Feuchtigkeitsverteilung als auch für den Verlauf der fortschreitenden Verdampfungsfront ermittelt. Dem Trocknen des feuchten Körpers in der Periode abnehmender Geschwindigkeit und dem intensiven Trocknen durch molekulare Übertragung im Verdampfungsbereich entsprechend wurden zwei mathematische Modelle betrachtet.

Es wird gezeigt, daß das in [1] gelöste Problem ein sehr spezieller Fall der hier vorgelegten Lösung ist. Der Einfluß einiger der dimensionslosen Parameter wird durch Beispiele erläutert.

ТОЧНОЕ РЕШЕНИЕ ЗАДАЧИ О РАСПРЕДЕЛЕНИИ ТЕМПЕРАТУРЫ И ВЛАЖНОСТИ В ПОРИСТОМ ПОЛУПРОСТРАНСТВЕ С ДВИЖУЩИМСЯ ФРОНТОМ ИСПАРЕНИЯ

Аннотация — Получены точные распределения температуры и влажности, а также положение движущегося фронта испарения в пористом полупространстве. Рассматриваются две математические модели, соответствующие сушке влажного тела при уменьшающейся скорости сушки и интенсивной сушке с молярным переносом в области испарения. Показано, что задача, решенная в [1], представляет частный случай данного решения. Влияние некоторых безразмерных параметров иллюстрируется примерами.